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## Symmetric Key Encryption

## Crypto - a brief introduction

- Cryptology — The art and science of making and breaking "secret codes"
- Cryptography — making "secret codes"
- ychrpyaprtgo
- $\mathrm{C}=\mathrm{M} \oplus \mathrm{K}$
- Cryptanalysis — breaking "secret codes"
- ychrpyaprtgo is cracked to $\qquad$ , QED.
- Crypto - all of the above (and more)
- More on non-repudiation (signature), authentication, identification, zero-knowledge, commitment, and more...
- Any reference books?... Bruce Schneier, HAC

A cipher or cryptosystem is used for encrypting/decrypting a plaintext/ciphertext


## Cryptanalysis

## Basic assumption

- Known as Kerckhoffs Principle
- The system is completely known to the attacker
- Only the key is secret
- Crypto algorithms are not secret
- No "security through obscurity"

Objective of an attacker

- Identify secret key used to encrypt a ciphertext
- (OR) recover the plaintext of a ciphertext without the secret key

Examples of (Classical) Symmetric Key Encryption Algorithms Classical Cryptography

Ciphertexts:

1. IRXUVFRUHDAGVHYHABHDUVDIR
2. VSRQJHEREVTXDUHSDQWU

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $j$ | $k$ | $I$ | $m$ | $n$ | $o$ | $p$ | $q$ | $r$ | $s$ | $t$ | $u$ | $v$ | $w$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Simple Substitution: each plaintext letter is substituted by a distinct ciphertext letter

## EIMBULJIWLNYANJMVLIURAHIWAI



DEPARTMENTOFCOMPUTERSCIENCE

- What's the ciphertext of "solutionstofinalexam"?

An example of simple substitution... Copyright 2002 by Randy Glasbergen. www.glasbergen.com

"Encryption software is expensive...so we just rearranged all the letters on your keyboard."

## An Example

Ciphertext (encrypted using simple substitution) PBFPVYFBQXZTYFPBFEQJHDXXQVAPTPQJKTOYQWIPBVWLXTOXBTFXQWAX BVCXQWAXFQJVWLEQNTOZQGGQLFXQWAKVWLXQWAEBIPBFXFQVXGTVJ VWLBTPQWAEBFPBFHCVLXBQUFEVWLXGDPEQVPQGVPPBFTIXPFHXZHVFA GFOTHFEFBQUFTDHZBQPOTHXTYFTODXQHFTDPTOGHFQPBQWAQJJTODX QHFOQPWTBDHHIXQVAPBFZQHCFWPFHPBFIPBQWKFABVYYDZBOTHPBQP QJTQOTOGHFQAPBFEQJHDXXQVAVXEBQPEFZBVFOJIWFFACFCCFHQWAUV WFLQHGFXVAFXQHFUFHILTTAVWAFFAWTEVOITDHFHFQAITIXPFHXAFQHEF ZQWGFLVWPTOFFA

# Question: how secure is Simple Substitution? 

Let's do some analysis...

- A secret key (in Simple Substitution) is a random permutation of the alphabetic characters.
- E.g.

| $a$ | $b$ | $c$ | $d$ | $e$ | $f$ | $g$ | $h$ | $i$ | $j$ | $k$ | $l$ | $m$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $X$ | $N$ | $Y$ | $A$ | $H$ | $P$ | $O$ | $G$ | $Z$ | $Q$ | $W$ | $B$ | $T$ |
| $n$ | $o$ | $p$ | $q$ | $r$ | $s$ | $t$ | $u$ | $v$ | $w$ | $x$ | $y$ | $z$ |
| $S$ | $F$ | $L$ | $R$ | $C$ | $V$ | $M$ | $U$ | $E$ | $K$ | $J$ | $D$ | $I$ |

- Each permutation is a potential candidate of the secret key
- Question: how many distinct permutations are there? (in other words, how many distinct secret keys are in the key space?)
- Total number of possible permutations

26!

- $26!=403,291,461,126,605,635,584,000,000(27$ digits $) \approx 2^{88}$
- Maybe... write a computer program to try all the possible keys exhaustively... (so-called Brute-force Attack)
- Calculation: suppose we have one million 3 GHz PCs which can try $\underline{3}$ billion permutations per second, the machines will take 4,263 years to try all the 26 ! permutations...
- Not so efficient
- Question: any better cracking algorithm?


## Statistical Attack / Character Frequency Attack

- An interesting observation on simple substitution: the relative letter frequencies do not change during encryption
- letters in an alphabet are not equally common
- in English, e and t are by far the most common letters
- Probability distribution of the 26 English letters (Beker and Piper, 1982)

| letter | probability | letter | probability |
| :---: | :---: | :---: | :---: |
| A | .082 | N | .067 |
| B | .015 | O | .075 |
| C | .028 | P | .019 |
| D | .043 | Q | .001 |
| E | .127 | R | .060 |
| F | .022 | S | .063 |
| G | .020 | T | .091 |
| H | .061 | U | .028 |
| I | .070 | V | .010 |
| J | .002 | W | .023 |
| K | .008 | X | .001 |
| L | .040 | Y | .020 |
| M | .024 | Z | .001 |

Basic Approach of Statistic Attack:
1.Identify possible encryptions of letter ' e ' (the most common English letter)
2.Identify possible diagrams starting/finishing with letter ' $e$ '
3.Use trigrams (e.g. find 'the')
4.Identify word boundaries
Iterate the above for the $2^{\text {nd }}$ most common English letter and so on.

Ciphertext:
PBFPVYFBQXZTYFPBFEQJHDXXQVAPTPQJKTOYQWIPBVWLXTOXBTFXQWAX BVCXQWAXFQJVWLEQNTOZQGGQLFXQWAKVWLXQWAEBIPBFXFQVXGTVJV WLBTPQWAEBFPBFHCVLXBQUFEVWLXGDPEQVPQGVPPBFTIXPFHXZHVFAG FOTHFEFBQUFTDHZBQPOTHXTYFTODXQHFTDPTOGHFQPBQWAQJJTODXQH FOQPWTBDHHIXQVAPBFZQHCFWPFHPBFIPBQWKFABVYYDZBOTHPBQPQJT QOTOGHFQAPBFEQJHDXXQVAVXEBQPEFZBVFOJIWFFACFCCFHQWAUVWFL QHGFXVAFXQHFUFHILTTAVWAFFAWTEVOITDHFHFQAITIXPFHXAFQHEFZQW GFLVWPTOFFA

## Ciphertext frequency counts:

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 21 | 26 | 6 | 10 | 12 | 51 | 10 | 25 | 10 | 9 | 3 | 10 | 0 | 1 | 15 | 28 | 42 | 0 | 0 | 27 | 4 | 24 | 22 | 28 | 6 | 8 |

Question: How to build a symmetric key cryptosystem which is secure against statistical attack?

## One-time Pad Encryption

## Encryption: Plaintext $\oplus$ Key $=$ Ciphertex $\dagger$

```
\[
\mathrm{e}=000 \mathrm{~h}=001 \mathrm{i}=010 \mathrm{k}=011 \mathrm{l}=100 \mathrm{r}=101 \mathrm{~s}=110 \mathrm{t}=111
\]
```

h e i l h i t l e r
Plaintext: $\begin{array}{llllllllll}001 & 000 & 010 & 100 & 001 & 010 & 111 & 100 & 000 & 101\end{array}$
Key: $1111 \begin{array}{llllllllll}101 & 110 & 101 & 111 & 100 & 000 & 101 & 110 & 000\end{array}$
Ciphertext: $110 \begin{array}{llllllllll}101 & 100 & 001 & 110 & 110 & 111 & 001 & 110 & 101\end{array}$

$$
s \quad r \quad l \quad h \quad s \quad s \quad t \quad h \quad s \quad r
$$

```
e=000 h=001 i=010 k=011 l=100 r=101 s=110 t=111
```


## Decryption: Ciphertext $\oplus$ Key = Plaintex $\dagger$



Ciphertext: $110 \quad 101 \quad 100 \quad 001 \quad 110 \quad 110 \quad 111 \quad 001$
$\begin{array}{rllllllllll}\text { Key: } & 111 & 101 & 110 & 101 & 111 & 100 & 000 & 101 & 110 & 000 \\ \text { Plaintext: } & 001 & 000 & 010 & 100 & 001 & 010 & 111 & 100 & 000 & 101\end{array}$
h e i l h i t l e r

- Pad must be random, used only once
$\square$ Pad has the same size as message

Questions: What are the current symmetric key cryptosystems?
There are many...
They can be categorized into two types:
1.Stream Cipher
2.Block Cipher

## Stream Ciphers



- Secret key length: 128 bits, 256 bits, etc.
- Maximum plaintext length: usually can be arbitrarily long.
- Security: Given a "long" segment of keystream (e.g. $2^{40}$ bits), the secret key cannot be derived AND the subsequent segment of the keystream cannot be deducted.


## RC4

- A stream cipher
- Ron's code version 4 (Ronald Rivest)
- Stream ciphers are generally faster than block ciphers
- RC4
- Stage 1: RC4 initialization
- Stage 2: RC4 keystream generation


## RC4 Initialization

- Setup:
byte key[N]; // secret key (e.g. $N=16$, i.e. 128-bit key)
byte K[256]; // keying material
byte S[256]; // internal states
- Initialization:

$$
\begin{aligned}
& \text { for } i=0 \text { to } 255 \\
& \quad \begin{array}{l}
S[i]=i \\
\\
\quad K[i]=\operatorname{key}[i(\bmod N)] \\
j=0 \\
\text { for } i=0 \text { to } 255 \\
\quad j=(j+S[i]+K[i]) \bmod 256 \\
\quad \operatorname{swap}(S[i], S[j]) \\
i=j=0
\end{array}
\end{aligned}
$$

- $S[]$ is the permutation of $0,1, \ldots, 255$


## RC4 Keystream Generation

- To output a keystream byte, swap table elements and select a byte
- Keystream generation:

```
    i = (i + 1) mod 256
    j = (j + S[i]) mod 256
    swap(S[i], S[j])
    t = (S[i] + S[j]) mod 256
    KeyStreamByteSelected = S[t]
```

- Use the KeyStreamByteSelected to do XOR with one byte of plaintext, then iterate the keystream generation steps above for getting another byte of keystream
- Note: Some research results show that the first 256 bytes must be discarded, otherwise attacker may be able to recover the key.


## Block Ciphers



- A block cipher takes a block of plaintext and a secret key, produces a block of ciphertext.
- The key is reused for different plaintext blocks
- Typical block sizes: 64 bits, 128 bits, 192 bits, 256 bits
- Key sizes: 56 bits (DES), 128/192/256 bits (AES)
- Popular block ciphers: DES, 3DES, AES, Twofish, Serpent


## (Iterated) Block Cipher

- Ciphertext obtained from plaintext by iterating a round function
- Input to round function consists of key and the output of previous round
- DES: 16 rounds of Feistel round function
- Block size: 64 bits

- Key size: 56 bits




## Security of DES

- Security of DES depends solely on the internals of $f$
- Thirty years of intense analysis has revealed no "back door"
- The most effective attack today against DES is still the exhaustive key search (aka bruteforce attack)


## Exhaustive Key Search

- Given a plaintext $x$ and corresponding ciphertext $y$, every possible key would be tested until a key K is found such that

$$
E(K, x)=y
$$

Note: there may be more than one such key K.

- For DES, total number of keys $=2^{56} \approx 7.2 \times 10^{16}$ keys
- Assume at the speed of $10^{6}$ encryptions per second, it would need more than 1000 years to break DES.
- Diffie and Hellman postulated in 1977 that a DES cracking machine with $10^{6}$ processors, each could test $10^{6}$ keys per second, could be built for about US\$20M.
- This machine can break DES in about 10 hours.


## Exhaustive Key Search

- In 1993, Michael Wiener presented a pipelined chip which tests $5 \times 10^{7}$ DES keys per second.
- Each chip could cost US $\$ 10$ and a frame of 5760 chips would cost about $\$ 100 \mathrm{~K}$.

| Machine Unit Cost | Expected Time |
| :--- | :--- |
| $\$ 100,000$ (1 frame) | 35 hours |
| $\$ 1,000,000$ (10 frames) | 3.5 hours |
| $\$ 10,000,000$ (100 frames) | 21 minutes |

- In 1998, DES cracker (nicknamed "Deep Crack" http://en.wikipedia.org/wiki/EFF DES cracker) was built by the Electronic Frontier Foundation (EFF).
- It performs 256 DES operations in 56 hours.
- Cost: US\$250K (first piece), US\$50K - \$75K (duplicates).
- Software version of DES cracking effort can be found at http://www.distributed.net/des/


## What Should We Use Today?

- 3DES (or Triple DES)
- AES (or Rijndael)
- Other candidates
- Twofish
- RC6
- Serpent


## Triple DES and DESX

- Triple DES: two 56-bit keys

- DESX: three keys

$$
\mathrm{C}=\mathrm{K}_{3} \oplus \mathrm{DES}\left(\mathrm{~K}_{2}, \mathrm{M} \oplus \mathrm{~K}_{1}\right)
$$



- Similar security to DES using differential cryptanalysis and linear cryptanalysis, which are theoretical attacks
- But much harder to break using exhaustive key search than DES What are the sizes of $K 1, K 2$, and $K 3$ ?


## Advanced Encryption Standard

- Replacement for DES
- AES competition (late 90's)
- NSA openly involved
- Many strong algorithms were proposed and cryptanalyzed publicly
- Rijndael Algorithm was ultimately selected
- Pronounced like "Rain Doll" or "Rhine Doll"
- Iterated block cipher (like DES)
- Not using Feistel round function (unlike DES)


## AES (Advanced Encryption Standard)

- Replacement of DES
- Block size: 128 bits
- Key length: 128, 192 or 256 bits (independent of block size)
- 10 to 14 rounds (depends on key length)
- Each round uses 4 functions (in 3 "layers")
- ByteSub (nonlinear layer)
- ShiftRow (linear mixing layer)
- MixColumn (nonlinear layer)
- AddRoundKey (key addition layer)



## Key Space

- The Key Space of a cipher is the set of all possible and distinct secret keys
- E.g. The key space of DES is all distinct 56-bit binary strings
- E.g. The size of the key space of simple substitution for case-insensitive English alphabet is 26!
- What's the key space size of AES?
- What's the key space size of RC4?


## Multiple Blocks



- How to encrypt multiple blocks?
- A new key for each block?
- As bad as (or worse than) the one-time pad!
- Encrypt each block independently?
- Make encryption depend on previous block(s), i.e., "chain" the blocks together?
- How to handle partial blocks?


## Modes of Operation

- Many modes of operation - we discuss three
- Electronic Codebook (ECB) mode
- Obvious thing to do
- Encrypt each block independently
- There is a serious weakness
- Cipher Block Chaining (CBC) mode
- Chain the blocks together
- More secure than ECB
- Counter Mode (CTR) mode
- Acts like a stream cipher
- Popular for random access


## ECB Mode

- Notations: $\mathrm{C}=\mathrm{E}(\mathrm{K}, \mathrm{P}) \quad \mathrm{P}=\mathrm{D}(\mathrm{K}, \mathrm{C})$
- Given plaintext $\mathrm{P}=\mathrm{P}_{0}, \mathrm{P}_{1}, \ldots, \mathrm{P}_{\mathrm{m}}, \ldots$ (in blocks)
- Obvious way of using a block cipher is to encrypt plaintext blocks independently


## Encrypt

$$
\begin{aligned}
& \mathrm{C}_{0}=\mathrm{E}\left(\mathrm{~K}, \mathrm{P}_{0}\right), \\
& \mathrm{C}_{1}=\mathrm{E}\left(\mathrm{~K}, \mathrm{P}_{1}\right), \\
& \mathrm{C}_{2}=\mathrm{E}\left(\mathrm{~K}, \mathrm{P}_{2}\right), \ldots
\end{aligned}
$$

Decrypt

$$
\begin{aligned}
& \mathrm{P}_{0}=\mathrm{D}\left(\mathrm{~K}, \mathrm{C}_{0}\right), \\
& \mathrm{P}_{1}=\mathrm{D}\left(\mathrm{~K}, \mathrm{C}_{1}\right), \\
& \mathrm{P}_{2}=\mathrm{D}\left(\mathrm{~K}, \mathrm{C}_{2}\right), \ldots
\end{aligned}
$$



## ECB Cut and Paste Attack

- Suppose plaintext is

Alice digs Bob. Trudy digs Tom.

- Assuming 64-bit blocks and 8-bit ASCII:

$$
\begin{aligned}
& \mathrm{P}_{0}=" \text { Alice di" } \mathrm{P}_{1}=" g s \text { Bob. ",", } \\
& \mathrm{P}_{2}=" \text { "rudy di", } \mathrm{P}_{3}=" g s \text { Tom. " }
\end{aligned}
$$

- Ciphertext: $\mathrm{C}_{0}, \mathrm{C}_{1}, \mathrm{C}_{2}, \mathrm{C}_{3}$
- Trudy cuts and pastes: $\mathrm{C}_{0}, \mathrm{C}_{3}, \mathrm{C}_{2}, \mathrm{C}_{1}$
- Decrypts as
Alice digs Tom. Trudy digs Bob.


## ECB Weakness

- Suppose $\mathrm{P}_{\mathrm{i}}=\mathrm{P}_{\mathrm{j}}$
- Then $\mathrm{C}_{\mathrm{i}}=\mathrm{C}_{\mathrm{j}}$ and Trudy knows $\mathrm{P}_{\mathrm{i}}=\mathrm{P}_{\mathrm{j}}$
- This gives Trudy some information, even if she does not know $\mathrm{P}_{\mathrm{i}}$ or $\mathrm{P}_{\mathrm{j}}$
- Is this a serious issue?


## Alice Hates ECB Mode

- Alice's uncompressed image, Alice ECB encrypted

- Why does this happen?
- Same plaintext block $\Rightarrow$ same ciphertext!


## CBC Mode

- Blocks are "chained" together
- A random initialization vector, or IV, is required to initialize CBC mode
- IV is random, but is not a secret


## Encryption

$\mathrm{C}_{0}=\mathrm{E}\left(\mathrm{K}, \mathrm{IV} \oplus \mathrm{P}_{0}\right)$,
$\mathrm{C}_{1}=\mathrm{E}\left(\mathrm{K}, \mathrm{C}_{0} \oplus \mathrm{P}_{1}\right)$,
$\mathrm{C}_{2}=\mathrm{E}\left(\mathrm{K}, \mathrm{C}_{1} \oplus \mathrm{P}_{2}\right), \ldots$

## Decryption

$$
\begin{aligned}
& \mathrm{P}_{0}=\mathrm{IV} \oplus \mathrm{D}\left(\mathrm{~K}, \mathrm{C}_{0}\right), \\
& \mathrm{P}_{1}=\mathrm{C}_{0} \oplus \mathrm{D}\left(\mathrm{~K}, \mathrm{C}_{1}\right), \\
& \mathrm{P}_{2}=\mathrm{C}_{1} \oplus \mathrm{D}\left(\mathrm{~K}, \mathrm{C}_{2}\right), \ldots
\end{aligned}
$$


$C_{0}$


## Alice Likes CBC Mode

- Alice's uncompressed image, Alice CBC encrypted

- Why does this happen?
- Same plaintext yields different ciphertext!


## Counter Mode (CTR)

- Use block cipher like stream cipher


## Encryption

$\mathrm{C}_{0}=\mathrm{P}_{0} \oplus \mathrm{E}(\mathrm{K}, \mathrm{IV})$,
$\mathrm{C}_{1}=\mathrm{P}_{1} \oplus \mathrm{E}(\mathrm{K}, \mathrm{IV}+1)$,
$\mathrm{C}_{2}=\mathrm{P}_{2} \oplus \mathrm{E}(\mathrm{K}, \mathrm{IV}+2), \ldots$

Decryption
$\mathrm{P}_{0}=\mathrm{C}_{0} \oplus \mathrm{E}(\mathrm{K}, \mathrm{IV})$,
$P_{1}=C_{1} \oplus E(K, I V+1)$,
$\mathrm{P}_{2}=\mathrm{C}_{2} \oplus \mathrm{E}(\mathrm{K}, \mathrm{IV}+2), \ldots$

- CTR is good for random access (both READ and WRITE)
- CBC is good for random READ only, but not WRITE



## Summary

- Kerckhoffs Principle
- Simple Substitution Encryption and statistical attack
- One-time Pad Encryption
- Stream Cipher: RC4
- Block Cipher: DES, AES
- Key Space
- Modes of Operation

